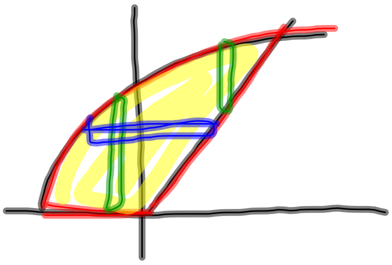
7.3 Volumes

solids of revolution: region, rotate about axis

solids of known cross section: base, cross section

solids of revolution < vertical rectangle dx / horizontal rectangle dy



horizontal is better here.

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disks $\int_a^b \pi r^2\,dx$ or $\int_c^d \pi r^2\,dy$

washers $\int_a^b \pi R^2 - \pi r^2\,dx$ or $\int_c^d \pi R^2 - \pi r^2\,dy$

shells $\int_a^b 2\pi r h\,dx$ or $\int_c^d 2\pi r h\,dy$

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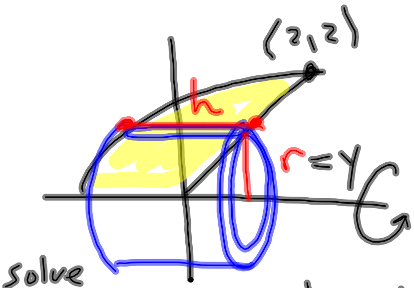
region bounded by $y=\sqrt{x+2}$, $y=x$, x-axis

rotated about: $x=y^2-2$ $x=y$

a) x-axis



$$\int_0^2 2\pi r h \, dy$$

$$v=\int_0^2 2\pi y\cdot\left(y-(y^2-2)\right)dy$$

$$v=\frac{16\pi}{3}$$

solve

$\sqrt{x+2}=x$

$x=2$

$y=2$

$h=y-(y^2-2)$

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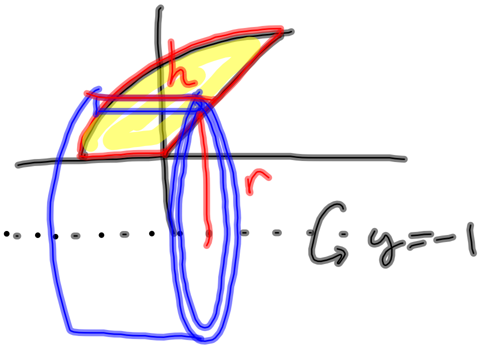
b) $y=-1$



$$\int_0^2 2\pi(y+1)\left(y-(y^2-2)\right)dy$$

$r=y+1$

$h=y-(y^2-2)$

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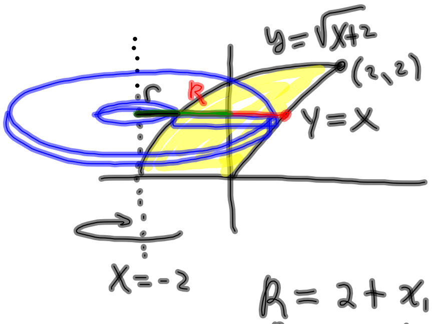
c) $x = -2$



$$\int_0^2 \pi R^2 - \pi r^2 \, dy$$

$$\int_0^2 \pi (2+y)^2 - \pi (y^2)^2 \, dy$$

$$= \frac{184\pi}{15}$$

$R = 2 + x_1$

$R = 2 + y$

$r = 2 + x_2$

$r = 2 + y^2 - 2$

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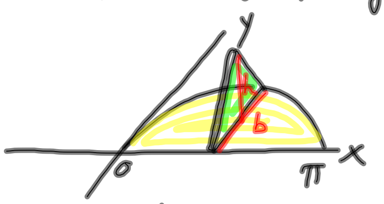
Solid of known cross section

base: region bounded by $y = \sin x$, x-axis $[0, \pi]$

cross sections $\perp$ x-axis (dx) are equilateral triangles with base from the x-axis to $y = \sin x$



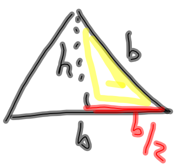
$\int$ area of cross section $\begin{matrix} dx \\ dy \end{matrix}$

$$\int_0^{\pi} \tfrac{1}{2} b h \, dx$$

$b = y = \sin x$

$h = \frac{\sqrt{3}}{2} b = \frac{\sqrt{3}}{2} \sin x$

$$\int_0^{\pi} \tfrac{1}{2} \sin x \cdot \tfrac{\sqrt{3}}{2} \sin x$$



$h^2 + \left(\frac{b}{2}\right)^2 = b^2$

$$= \frac{\pi\sqrt{3}}{8}$$

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